

# Lecture 2: Wigner-Eckart Theorem and Coupling Three or more Angular Momenta

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## 1. Review of Coupling Two Angular Momenta

- Quick reminder of Clebsch-Gordan coefficients.
- Brief discussion of how two angular momenta are added to form total angular momentum  $J$ .
- Key equations:

$$J_z|j, m\rangle = m|j, m\rangle \quad (1)$$

$$J_{\pm}|j, m\rangle = \sqrt{j(j+1) - m(m \pm 1)}|j, m \pm 1\rangle \quad (2)$$

- Role of Clebsch-Gordan coefficients as the basis for forming the coupled basis.

## 2. Rotation of Angular Momentum States and Operators

- Define rotation operator  $\hat{R}(\alpha, \beta, \gamma)$  using Euler angles.
- Action of rotation on states:

$$\hat{R}(\alpha, \beta, \gamma)|j, m\rangle = \sum_{m'} D_{m'm}^j(\alpha, \beta, \gamma)|j, m'\rangle$$

- Action on angular momentum operators:

$$\hat{R}(\alpha, \beta, \gamma)\hat{J}_i\hat{R}^{-1}(\alpha, \beta, \gamma) = \sum_j R_{ij}\hat{J}_j$$

## 3. Introduction to Irreducible Tensor Operators

- Define irreducible tensor operators  $T_q^{(k)}$  and discuss their transformation under rotations:

$$\hat{R}(\alpha, \beta, \gamma)T_q^{(k)}\hat{R}^{-1}(\alpha, \beta, \gamma) = \sum_{q'} D_{q'q}^k(\alpha, \beta, \gamma)T_{q'}^{(k)}$$

- Discuss the connection between angular momentum operators and tensor operators.
- Worked example: Transformation of a quadrupole moment operator.

## 4. Wigner-Eckart Theorem

- Statement of the theorem:

$$\langle j', m'|T_q^{(k)}|j, m\rangle = \langle j'||T^{(k)}||j\rangle \cdot \langle j', m'|k, q; j, m\rangle$$

- Explanation of separating the angular dependence from the reduced matrix element.

- Worked examples.

## 5. Coupling Three Angular Momenta

- Extending the coupling of angular momenta:

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3$$

- Introduction to recoupling coefficients and 6-j symbols.
- Example of a three angular momenta system and the use of recoupling coefficients.

## 6. Coupling Four Angular Momenta

- Extending the coupling of angular momenta:

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3 + \vec{J}_4$$

- Introduction to recoupling coefficients and 9-j symbols.

## 7. Applications to Physical Systems

- Example: Selection rules for atomic transitions (dipole/quadrupole). (Prelude to the next Lecture on Multipole (E1, E1 & M1, M2, M3) Transitions.)